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ENDOGENOUS OVERLAPPING MARKET AREAS IN A SPATIAL DUOPOLY

by

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RÉSUMÉ

Nous considérons dans ce texte un duopole spatial en prix et aires de marché. Une entreprise doit informer les consommateurs sur son produit et son prix; les consommateurs ne furètent pas. Choisir la proportion de consommateurs à informer, c'est choisir son aire de marché. Les localisations des entreprises sont données. Nous caractérisons et comparons les équilibres de Stackelberg et les équilibres de Bertrand. Nous montrons que si le coût d'information est faible, un duopoleur choisit d'informer tous les consommateurs dans les équilibres de Bertrand (donc, aires de marché séparées) alors que le leader dans l'équilibre de Stackelberg peut décider de *ne pas* informer tous les consommateurs si la concurrence duopolistique est suffisamment vive (entraînant ainsi des aires de marché imbriquées). De plus, la prise en compte de la nature stratégique des choix d'aires de marché modifie de façon sensible les relations d'équilibre entre la différentiation, les profits et les marges.

Mots clés : Couverture stratégique, marchés imbriqués, équilibre de Stackelberg.

ABSTRACT

We consider in this paper competition through prices and market coverages in a spatial duopoly. A firm must inform consumers of its product and price and consumers do not search; choosing the proportion of consumers to inform is choosing market coverage. The locations of firms are given. We characterize and compare the Stackelberg equilibria and the Bertrand (full market coverage) equilibria. We show that when the cost of informing the consumers is small, a duopolist always chooses to inform all consumers in the Bertrand equilibrium (generating separate markets), but the leader in the Stackelberg equilibrium may choose *not* to inform everyone when the duopolistic competition is intense enough (thereby generating overlapping markets). Moreover, considering market coverage as chosen by the firm with a significant strategic component (through the Stackelberg structure of the market game) changes in important ways the equilibrium relationship between the level of differentiation, profits and markups.

Key words : Strategic coverage, Overlapping markets, Stackelberg equilibrium.

1. INTRODUCTION

In the d'Aspremont, Gabszewicz and Thisse (1979) reformulation of the canonical spatial competition model of Hotelling (1929), the market areas of the firms do not overlap.[1] In order to have overlapping market areas it must be postulated either that competition is not a price competition but rather a Cournot like quantity competition as for example in Weskamp (1985),[2] Anderson and Neven (1989) or Hamilton, Thisse and Weskamp (1989), or that the goods marketed by each firm have some specific characteristics other than the sole location of the firm selling them as for example in de Palma, Ginsburgh, Papageorgiu and Thisse (1985).[3] In all these formulations, either the two sides of the market are perfectly informed or the imperfection of information does not play a crucial role in the determination of the equilibrium. In de Palma et al. (1985), the firms know the distribution of consumer tastes at each point of the space and it is a sufficient statistics to determine the price policies of the firms.

More recently Gabszewicz, Garella and Nollet (1988) extending the work of Gabszewicz and Garella (1986) have relaxed the assumption of perfect information of both the consumers and the firms regarding the other side of the market. In their model the consumers know ab initio the price quoted by the nearest seller but not the price quoted by the other firms. Hence the partially uninformed consumers must decide whether they will bear the cost of getting additional information or not. They show that at symmetric equilibria the firms quote the highest prices which induce no search by any consumer so that the respective markets do not overlap. With asymmetrically (and exogenously) located firms there exist price equilibria where some consumers will search for more information but even so the markets do not overlap.

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[1] See Gabszewicz and Thisse (1986) and Greenhut, Norman and Hung (1987) for recent surveys of location theory.

[2] Weskamp's model is a "network" type model; but the overlapping property holds in the sense that for some nodes more than one firm have non zero sales.

[3] See also Anderson and de Palma (1988) and Anderson, de Palma and Thisse (1989) for more on this subject.

Schultz and Stahl (1988) develop a model of consumer search over variety and price. Firms choose their location, variety and price and in equilibrium, more than one variety may be offered at one given location and a variety may be offered at more than one location. The consumer preferences are modelled so that more variety means increased demand: a consumer visits only one market place and therefore a visit is more desirable when more variants are offered at that market place. Each consumer is assumed to know, before searching, the location and size (number of varieties offered) of market places; he forms a (true) point forecast of the expected price charged at a given market place and also a forecast of the characteristics of the varieties. The authors consider and compare the perfect Nash equilibria of two market structures: a large number of potential one-product firms, each choosing in a three-stage game to enter or not (in stage 1; an entrant must incur an entry cost) and upon entry, a plant location (stage 2) and both a product variety and a price (stage 3); a one-plant monopolist deciding on varieties and prices. They show that if consumers are choosy enough, then all one-product firms locate at the same location (in any case, there is always two or more firms at each equilibrium location) and may charge prices in excess of that charged by the multiproduct monopolist with the same set of variants; they also provide conditions under which the latter offers as many varieties as the former: hence, monopoly may increase welfare.[4] But since search costs are function only of the distance to the marketplace, the overlapping of markets (not explicitly considered by the authors) will proceed from the same factors than in de Palma, Ginsburgh, Papageorgiu and Thisse (1985).

In a different context, Boyer, Laffont, Mahenc and Moreaux (1990) provide an exhaustive analysis, in the space of prior beliefs and cost differentials, of the nature of Perfect Bayesian D1-equilibria in a two stage model of sequential location choices (stage 1) and simultaneous delivered pricing schedules (stage 2) when the firms are asymmetrically

[4] Intuitively, those results may be understood as an application of the basic result that a m-product monopolist will sell at lower prices than m one-product oligopolists when the products are complements; in the model of Schultz and Stahl, the products are both substitutes (a consumer buys only one variety, if any) and complements (each variety offered at a given location increases the demand at that market place by making search more profitable, in expected terms).

informed in stage 1 about the production cost of their competitor;[5] but again, in the production stage the markets are separated.

But casual observation of markets reveals that they more often overlap than not; hence, it seems to us that the usual non-overlapping property of equilibria in most spatial competition models is a weak point of the theory.

One way to generate overlapping markets is to suppose that consumers are "born" unaware of the prices charged by the firms (and even of their existence), that they do not search but can become informed of the price charged by a firm if they receive a message from that given firm. In such a framework, each firm chooses not only its price but also the number (proportion) of consumers it will inform. It is this viewpoint of imperfect information that we take in this paper. We suppose that given locations (determined in a preceding stage of the game not modelled here), the firms must inform the consumers about their respective location and price. Hence the firms compete in both prices and "informative advertising"; we consider only the case of a duopoly. Overlapping markets can result from less than full information of all the potential buyers at the equilibrium. We compare two forms of duopoly markets. In the first case the firms move simultaneously while in the second case they move sequentially with one of the firms taking the leadership and the other accepting the followership. Using a one-parameter quadratic cost of information function, we show that at simultaneous Nash equilibria, overlapping never occurs when the cost of informing the consumers is small (although it does occur when the cost of information is high enough) whereas at Stackelberg equilibria the leader may choose not to inform all the potential buyers even if the cost of informing the consumers is zero, implying in this limit case overlapping market areas with a zero cost of information; a fortiori, when the cost of informing the consumers become significant, overlapping market areas appear. The main result of this paper is that when the cost to

[5] More specifically, the first-mover in stage 1 knows its marginal production cost (equal either to c or $c - \alpha$) when choosing its location but the second-mover whose marginal cost is always c can only observe the location chosen by the first-mover firm.

the firms of informing the consumers is "small", the duopolists always choose to inform all consumers in the Bertrand-Nash equilibria but the leader in the Stackelberg equilibria will choose under the same cost conditions *not* to inform everyone when the level of duopolistic competition is "intense enough", that is when the goods are "good enough" substitutes.

Moreover, considering a market coverage variable endogenously chosen by the firm with a significant strategic component (made possible here by the Stackelberg structure of the market game) will change in important ways the equilibrium relationships between the level of product differentiation and profits and markups. The two traditional measure of increased product differentiation – an increase in "transportation" costs and a reduction in the reservation price (value) – have different impacts on profits and markups. We believe that the present model not only brings into the economic analysis of spatial (product design) competition an important strategic variable, the decision on market coverage, but also provides a richer analysis of the implications of product differentiation.

Grossman and Shapiro (1984) have analysed the symmetric n-firm Nash equilibria of a model where firms are symmetrically located around a circular city and choose both a price and a level of informative advertising. Their advertising technology is such that it is infinitely costly to inform all the consumers; hence markets always overlap. Although the focus of their paper is different, we can interpret their results in the context of spatial competition with overlapping markets. We will present their model and results in relation to our own model and results in section 5 below.

The next section is devoted to introducing the model and section 3 presents the simultaneous move (Bertrand-Nash) equilibria when all consumers are aware of the existence and prices of both firms; we characterize in section 4 the best-reply function giving one firm's optimal price as a function of the other firm's price *and* partial market coverage level; the stage is then set for the characterization of the simultaneous move (Bertrand-Nash) equilibria with partial market coverage and the numerical evaluation of

the leader-follower (Stackelberg) equilibria when firms choose both their price and their coverage of the market. In section 5, we conclude by discussing the related literature on informative advertising and "rationing".

2. THE MODEL

We consider the following context adapted from Butters (1977) and Tirole (1988). Assume a linear city with two firms located at the end points of the city, one firm, say firm 1, at $x = 0$ and one firm at $x = 1$. The consumers are distributed uniformly on this interval $[0, 1]$ and each one demands inelastically one unit of the homogeneous good if the cost is below a reservation price s. In addition to paying the price set by the firm, a consumer located at x_1 must bear quadratic transportation costs of tx_1^2 if he buys from firm 1 and $t(1-x_1)^2$ if he buys from firm 2 located at $x = 1$: the goods are differentiated by their locations only.[6] Suppose that at $x \in (0,1)$ the consumer is indifferent between buying from firm 1 or firm 2, that is

$$p_1 + tx^2 = p_2 + t(1-x)^2 \leq s, \tag{2.1}$$

and suppose that all consumers are aware of the existence of both firms. All consumers to the left of x will buy from firm 1 and all those to the right of x will buy from firm 2. In this context, the demand facing firm i may be written as ("a" stands for "aware of the existence of the other firm"):

$$d_i^a(p_1, p_2) = \begin{cases} 1 & \text{if} \quad p_i \leq p_j - t \\ x = (p_j - p_i + t)/(2t) & \text{if} \quad p_j - t \leq p_i \leq p_j + t \\ 0 & \text{if} \quad p_j + t \leq p_i. \end{cases} \tag{2.2}$$

[6] There are two main reasons for using quadratic transportation costs rather that linear ones. First, d'Aspremont et al. (1979) show that maximal product differentiation (the two firms located at the end points of a linear city, that is at $x = 0$ and $x = 1$ respectively) *results* from quadratic transportation costs in a model of location choice; hence, although we consider here those end point locations as exogenous, they are compatible (in a subgame perfect sense) with quadratic transportation costs. Second, quadratic transportation costs will generate continuous residual demand functions and continuous and concave profit functions contrary to the case of linear transportation costs.

The residual demand function facing firm i given p_j and assuming that all consumers are aware of both firms will more generally depend also on s since it may well be the case that for some location the price charged by one firm or both plus the quadratic transport cost exceeds the reservation price s. Moreover, if that is so in equilibrium, then some consumers may even not be served.[7]

Given p_j, the demand to firm i cannot be negative or larger than 1; it cannot be larger than the solution to $p_i + tx^2 = s$ [the consumer at such an x is just indifferent between buying the product from firm i and not buying at all] and it cannot be larger than $(p_j - p_i + t)/(2t)$ as seen above. Hence

$$d_i^a(p_1, p_2) = \max\{0, \min\{\frac{p_j - p_i + t}{2t}, \left(\frac{s - p_i}{t}\right)^{1/2}, 1\}\}. \tag{2.3}$$

The different cases can be illustrated as on figure 1 (a to d) where $f = \left(\frac{s-p_i}{t}\right)^{1/2}$ and $h = \left(\frac{p_j - p_i + t}{2t}\right)$.

[INSERT FIGURE 1 ABOUT HERE]

From figure 1 we can draw the demand itself as in figure 2 (for the case $s < t$ and $p_j < t$) where the straight line $C(x)$ of equation $p_i = p_j + t - 2tx$ is the solution to $p_i + tx^2 = p_j + t(1-x)^2$; curve $E(x)$ is the solution to $p_i + tx^2 = s$; $MRC(x)$ is the marginal revenue function defined from $C(x)$, that is $MRC(x) = p_j + t - 4tx$; $MRE(x)$ is the marginal revenue function defined from $E(x)$, that is $MRE(x) = s - 3tx^2$. Hence, the marginal revenue function $MR(x)$ for (2.3) is given by $MRE(x)$ for $x < x_1$ and by $MRC(x)$ for $x > x_1$; at $x = x_1$, the demand function is not differentiable. The value of x_1 is obtained given p_j from $p_j + t(1-x)^2 = s$, that is $x_1 = 1 - \left(\frac{s-p_j}{t}\right)^{1/2}$ and the value of $\hat{p}_i$ is obtained from $\hat{p}_i = s - tx_1^2 = p_j + t - 2tx_1$; replacing x_1 by its value, we obtain $\hat{p}_i = p_j - t + 2t\left(\frac{s-p_j}{t}\right)^{1/2}$; the value of x_2 is obtained from $MRC(x) = 0$, that is

[7] The reservation price s will play an important role in our model with a leader-follower structure; since we will show that the leader may choose not to inform all consumers while the follower does inform everyone, the profit of the follower (given that the leader is committed to its choice) would be unbounded if s is unbounded. Hence, we cannot simply assume that s is large enough that every consumer, aware of at least one firm, will buy in equilibrium.

$x_2 = (p_j + t)/(4t)$; the value of x_3 is obtained from $MRE(x) = 0$, that is $x_3 = \left(\frac{s}{3t}\right)^{1/2}$; the value of x_4 is obtained from $p_i = 0$ in the first branch of the demand (2.3), that is $x_4 = \frac{p_j+t}{2t}$; and finally the value of x_5 is similarly obtained from the second branch, that is $x_5 = (s/t)^{1/2}$.

[INSERT FIGURE 2 ABOUT HERE]

The demand function (2.3) can be rewritten (see figure 2 but recall that it is drawn with $s < t$ and $p_j < t$ hence both x_4 and x_5 are less than 1 but of course we could have both $x_4 > 1$, if $p_j < t$, and/or $x_5 > 1$, if $s > t$) as:

$$d_i^u(p_i, p_j) = \begin{cases} \frac{p_j+t-p_i}{2t} & \text{if} \quad p_i \leq \hat{p}_i \\ \left(\frac{s-p_i}{t}\right)^{1/2} & \text{if} \quad \hat{p}_i \leq p_i \leq s \\ 0 & \text{if} \quad s \leq p_i \end{cases} \tag{2.4}$$

and the inverse demand function can be written as a function of x as follows:

$$p_i(x, p_j) = \begin{cases} s - tx^2 & \text{if} \quad x \leq x_1 \\ p_j + t - 2tx & \text{if} \quad x_1 \leq x \leq x_4 \\ 0 & \text{if} \quad x_4 \leq x. \end{cases} \tag{2.5}$$

It is interesting to note that the demand function has a kink of Stigler type at $(x_1, \hat{p}_i)$. Hence the marginal revenue function $MR(x)$ is discontinuous at $x = x_1$. As p_j increases, the kink moves to the right on the $E(x)$ curve since that curve is independent of p_j while the $C(x)$ line moves up – or to the right – linearly with p_j.

Consider again figure 2 and the demand (2.4). These were derived for a consumer aware of the existence of both firms. A consumer located at x and aware of the existence of firm i only demands one unit of good i if $p_i + tx^2 \leq s$. Hence if all ($N = 1$) consumers are aware of firm i only, the demand for good i would be given by ("u" stands for "unaware of the existence of the other firm"): $d_i^u(p_i) = \left(\frac{s-p_i}{t}\right)^{1/2}$ and the inverse demand function would be given by: $p_i(x) = s - tx^2$, that is the demand is curve $E(x)$. Therefore, if $s \leq t$ [or $x_5 \leq 1$], then

$$d_i^u(p_i) = \begin{cases} \left(\frac{s}{t}\right)^{1/2} & \text{if} \quad p_i = 0 \\ \left(\frac{s-p_i}{t}\right)^{1/2} & \text{if} \quad 0 \leq p_i \leq s \\ 0 & \text{if} \quad s \leq p_i \end{cases} \tag{2.6}$$

and the inverse demand function is

$$p_i(x) = \begin{cases} s - tx^2 & \text{if} \quad 0 \leq x \leq (s/t)^{1/2} \\ 0 & \text{if} \quad (s/t)^{1/2} \leq x. \end{cases} \tag{2.7}$$

If $s \geq t$ [or $x_5 \geq 1$], then

$$d_i^u(p_i) = \begin{cases} 1 & \text{if} \quad p_i \leq s - t \\ \left(\frac{s-p_i}{t}\right)^{1/2} & \text{if} \quad s - t \leq p_i \leq s \\ 0 & \text{if} \quad s \leq p_i \end{cases} \tag{2.8}$$

and the inverse demand function is

$$p_i(x) = s - tx^2 \quad \text{for} \quad x \in [0,1]. \tag{2.9}$$

3. CHARACTERIZATION OF BERTRAND EQUILIBRIA

If a proportion $k_j, 0 \leq k_j \leq 1$, of consumers are aware of the existence of firm j and if those k_j consumers are randomly located in the interval [0,1] then the residual demand facing firm i will be:

$$d_i(p_i, p_j, k_j) = k_j d_i^n(p_i, p_j) + (1 - k_j) d_i^u(p_i) \tag{3.1}$$

with $d_i^n(p_i, p_j)$ given by (2.4) and $d_i^u(p_i)$ given either by (2.6) or by (2.8). And the profit function of firm i, assuming a zero cost of information, can be written as:

$$\pi_i(p_i . k_i . p_j, k_j) = (p_i - c) k_i d_i(p_i, p_j, k_j). \tag{3.2}$$

Let us first characterize the profit maximizing price p_i^* as a function of p_j assuming that $k_i = k_j = 1$, that is assuming that all consumers are aware of the existence of both firms. Moreover, let us assume that $c \equiv 0$. We can refer to figure 2. The profit maximizing price $p_i^*(p_j; s, t)$ will be either at $MRC(x) = 0$ on the linear portion $C(x)$ of the demand function, or at the kink point where the linear portion $C(x)$ and the concave portion $E(x)$ intersect, or at $MRE(x) = 0$ on the concave portion of the demand function. As we saw

previously, the value of x_1 is $1+\left(\frac{s-p_j}{t}\right)^{1/2}$. The value of $MRC(x)$ is for $x = x_1$ given by $p_j + t - 4t\left[1 + \left(\frac{s-p_j}{t}\right)^{1/2}\right]$; therefore $MRC(x_1) = 0$ if $p_j^2 + 10tp_j + (9t^2 - 16ts) = 0$, an expression which has always a negative root and, if $s \geq \frac{9t}{16}$, a non-negative one [if $s < \frac{9t}{16}$, the value of $MRC(x)$ at $x = x_1$ is negative and hence p_i^* is already at the kink when $p_j = 0$]. When it exists, the non-negative root is given by $-5t + 4(t^2 + ts)^{1/2}$; hence let

$$\tilde{p}_j = \begin{cases} -5t + 4(t^2 + ts)^{1/2} & \text{if} \quad s \geq \frac{9}{16}t \\ 0 & \text{otherwise} \end{cases} \tag{3.3}$$

and so, for $p_j \geq \tilde{p}_j$,[8] the profit maximizing price p_i^* is at the kink value $\hat{p}_i$

$$\hat{p}_i = \begin{cases} p_j - t + 2t\left(\frac{s-p_j}{t}\right)^{1/2} & \text{if} \quad p_j > s - t \\ s & \text{otherwise} \end{cases} \tag{3.4}$$

and the demand is given by x_1 as long as $x_1 < x_3 = \left(\frac{s}{3t}\right)^{1/2}$, that is as long as the kink $\hat{p}_i$ is above $\frac{2}{3}s$, that is as long as $p_j < \bar{p}_j$ [with $\bar{p}_j$ defined from $\bar{p}_j + t(1 - x_3)^2 = s$ if $x_3 \leq 1$ — that is if $s \leq 3t$ — in which case $\bar{p}_j = s - t(1 - x_3)^2 = s - t\left(1 - \left(\frac{s}{3t}\right)^{1/2}\right)^2$, and $\bar{p}_j = s$ if $s \geq 3t$]. So let

$$\bar{p}_j = \begin{cases} \frac{2}{3}s - t + 2\left(\frac{st}{3}\right)^{1/2} & \text{if} \quad s \leq 3t \\ s & \text{otherwise.} \end{cases} \tag{3.5}$$

For $p_j \geq \bar{p}_j$, the profit maximizing p_i^* is constant at $s - tx_3^2 = \frac{2}{3}s$, unless of course $x_3 > 1$ in which case $p_i^* = s - t$.

For relatively low value of p_j, the best reply price p_i is given by $MRC(x) = 0$ unless $s < \frac{9}{16}t$ in which case $MRC(x)$ is already negative at the kink and therefore p_i^* is at the kink value $\hat{p}_i$; for intermediate values, it is given by the kink point $\hat{p}_i$ (itself a decreasing function of p_j); for relatively large p_j, that is above $\bar{p}_j$, it is given by $MRE(x) = 0$ in which case further increases in p_j have no effect on p_i^* which remains constant at $\frac{2}{3}s$. From (1.5) we have for the case illustrated in figure 2, $p_i(x_2, p_j) = (p_j + t)/2$, $p_i(x_1, p_j) = \hat{p}_i$,

[8] This value $\tilde{p}_j$ is always less than s since $-5t + 4(t^2 + ts)^{1/2} - s = (3t + s)^2 > 0$.

$p_i(x_3, p_j) = \frac{2}{3}s$. Hence

$$p_i^*(p_j; s, t) = \begin{cases} (p_j + t)/2 & \text{if} \quad 0 \le p_j \le \tilde{p}_j \\ \hat{p}_i(p_j) = p_j - t + 2t\left(\frac{s-p_j}{t}\right)^{1/2} & \text{if} \quad \tilde{p}_j \le p_j \le \bar{p}_j \\ \frac{2}{3}s & \text{if} \quad \bar{p}_j \le p_j \quad \text{and} \quad s \le 3t \\ s - t & \text{if} \quad \bar{p}_j \le p_j \quad \text{and} \quad s \ge 3t. \end{cases} \tag{3.6}$$

It can easily be checked that (3.6) is continuous. The first portion of (3.6) is linear and increasing with p_j, the second portion is decreasing and concave in p_j and finally the third portion is constant either at $\frac{2}{3}s$ or at $s - t$. It is interesting to note that for $p_j < \tilde{p}_j$, the prices are strategic complements (the reaction functions are upward-sloping) while they are strategic substitutes for $\tilde{p}_j < p_j < \bar{p}_j$ (the reaction functions are downward-sloping) and strategically independent for $\bar{p}_j < p_j$ (the reaction functions are flat).

INSERT FIGURE 3 ABOUT HERE

The position of $p_i^*(p_j; s, t)$ and of its twin counterpart $p_j^*(p_i; s, t)$ will determine the existence and characteristics of Bertrand-Nash equilibria. As t increases for a given s, the linear portion of the best-reply function moves up and $\tilde{p}_j$ and $\bar{p}_j$ both increase with t if $t < \frac{1}{3}s$ and both decrease with t if $t > \frac{1}{3}s$. We therefore can have 4 different configurations as represented on figure 4(a to d), depending on the relative values of t and s.

[INSERT FIGURE 4 ABOUT HERE]

We will be in case (a) if $p_i(\tilde{p}_j) < \tilde{p}_j$, that is if

$$\tilde{p}_j - t + 2t\left(\frac{s - \tilde{p}_j}{t}\right)^{1/2} < \tilde{p}_j,$$

which from (3.3) can be written as

$$\left(\frac{s + 5t - 4(t^2 + ts)^{1/2}}{t}\right)^{1/2} < \frac{1}{2},$$

a condition which is satisfied for $t < \frac{4}{5}s$. For larger values of t, the point $(\tilde{p}_j, p_i^*(\tilde{p}_j))$ is above the 45°-line and therefore the intersection with that line will be either in the concave

portion of the reaction function or in the flat portion, the former case corresponding to values of $t \in [\frac{4}{5}s, \frac{4}{3}s]$ and the latter for values $t \geq \frac{4}{3}s$.

It can be shown that the concave portion of the two reaction functions when intersecting are in fact overlapping. Consider $p_i^*(p_j)$ for p_j in the interval $[\tilde{p}_j, \bar{p}_j]$. From (3.6):

$$p_i^*(p_j) = p_j - t + 2t\left(\frac{s-p_j}{t}\right)^{1/2}.$$

Suppose that $p_i^*(p_j) \in [\tilde{p}_i, \bar{p}_i]$. Then the best-reply by firm j to $p_i = p_i^*(p_j)$ is similarly given by

$$p_j^*(p_i^*(p_j)) = p_i^*(p_j) - t + 2t\left(\frac{s-p_i^*(p_j)}{t}\right)^{1/2}$$

$$= p_j - t + 2t\left(\frac{s-p_j}{t}\right)^{1/2} - t + 2t\left(\frac{s-p_j+t-2t\left(\frac{s-p_j}{t}\right)^{1/2}}{t}\right)^{1/2}$$

$$= p_j - 2t\left[1 - \left(\frac{s-p_j}{t}\right)^{1/2} - \left(\frac{s-p_j+t-2t\left(\frac{s-p_j}{t}\right)^{1/2}}{t}\right)^{1/2}\right]$$

$$= p_j \quad \text{if and only if}$$

$$1 - \left(\frac{s-p_j}{t}\right)^{1/2} = \left[\frac{s-p_j+t-2t\left(\frac{s-p_j}{t}\right)^{1/2}}{t}\right]^{1/2}.$$

Raising both sides to the power 2, we get

$$1 + \left(\frac{s-p_j}{t}\right) - 2\left(\frac{s-p_j}{t}\right)^{1/2} = \left(\frac{s-p_j}{t}\right) + 1 - 2\left(\frac{s-p_j}{t}\right)^{1/2}$$

which proves that the concave portions when intersecting are in fact overlapping.

The borderline between case (b) and case (c) is obtained when there is a complete overlapping of the concave portions of the reaction functions, that is for $\tilde{p}_j(s,t) = \frac{2}{3}s$; from (3.3) we have such a case if $-5t + 4(t^2+ts)^{1/2} = \frac{2}{3}s$, that is if $t = \frac{19+4\sqrt{10}}{27}s \approx 0.978s$.

And finally we will have case (d) if $\overline{p}_j > \frac{2}{3}s$, that is from (3.5) if $s < \frac{3}{4}t$. The different cases in figure 4 were drawn with the following values:

case (a): $s = 17, t = 10 < \frac{4}{5}s$; hence, $\tilde{p}_j(s,t) = 15.7$ and $\overline{p}_j(s,t) = 16.4$;
the symmetric equilibrium is at $p^e = t = 10$

case (b): $s = 17, t = 15.3 = 0.9s$; hence, $\tilde{p}_j(s,t) = 12.4$ and $\overline{p}_j(s,t) = 14.7$;
the symmetric equilibrium is at $p^e = s - \frac{1}{4}t = 13.2$

case (c): $s = 17, t = 20.4 = 1.2s$; hence, $\tilde{p}_j(s,t) = 8.5$ and $\overline{p}_j(s,t) = 12.4$;
the symmetric equilibrium is at $p^e = s - \frac{1}{4}t = 11.9$

case (d): $s = 17, t = 25 > \frac{4}{3}s$; hence, $\tilde{p}_j(s,t) = 4.6$ and $\overline{p}_j(s,t) = 10.1$;
the symmetric equilibrium is at $p^e = \frac{2}{3}s = 11.3$

In the first case, we have a unique, symmetric and stable Bertrand-Nash equilibrium on the linear portion of the reaction functions. In the second and third cases, we have a *stable set of unstable equilibria* on the concave portion of the reaction functions indicating that both firms are then at the kink of their respective demand functions. Finally in the fourth case, we have a unique, symmetric and stable equilibrium on the constant portion of the reaction functions; in this last case, not all consumers buy the goods and both firms are at their monopolistic equilibrium. *The symmetric equilibrium price, as a function of t, is first increasing from $p^e = 0$, reaches a maximum at $p^e = \frac{4}{5}s$ reached for $t = \frac{4}{5}s$, then decreases with t to a minimum at $p^e = \frac{2}{3}s$ reached for $t = \frac{4}{3}s$ and then stays at that value for larger values of t.*

4. THE CHARACTERIZATION OF EQUILIBRIA IN THE (PRICE, MARKET COVERAGE) STRATEGY SPACE.

To allow a complete presentation of the different possibilities, we will assume that $\frac{1}{3}s < t < \frac{4}{5}s$ (that is $\frac{5}{4}t < s < 3t$), implying that $s - t < \frac{2}{3}s$ and that with full information ($k_i = k_j = 1$) the Bertrand equilibrium will be on the linear and increasing portion of the reaction function (3.6) represented in figure 3(a). The demand function for the case

$\frac{1}{3}s < t < \frac{4}{5}s$ is depicted on figure 5.

[INSERT FIGURE 5 ABOUT HERE]

$$DA: x = \begin{cases} \min\{1, \frac{p_j+t}{2t}\} & \text{if} \quad p_i \leq \max\{0, p_j - t\} \\ \frac{p_j+t}{2t} - \frac{1}{2t}p_i & \text{if} \quad \max\{0, p_j - t\} \leq p_i \leq \hat{p}_i \\ \left(\frac{s-p_i}{t}\right)^{1/2} & \text{if} \quad \hat{p}_i \leq p_i \\ 0 & \text{if} \quad s \leq p_i; \end{cases}$$

$$DU: x = \begin{cases} \min\{1, \left(\frac{s}{t}\right)^{1/2}\} & \text{if} \quad p_i \leq \max\{0, s - t\} \\ \left(\frac{s-p_i}{t}\right)^{1/2} & \text{if} \quad \max\{0, s - t\} \leq p_i \leq s \\ 0 & \text{if} \quad s \leq p_i; \end{cases}$$

$$MRDA: x = \begin{cases} \min\{1, \frac{p_j+t}{2t} - \frac{1}{t}p_i\} & \text{if} \quad p_i \leq \max\{0, p_j - t\} \\ \frac{p_j+t}{2t} - \frac{1}{t}p_i & \text{if} \quad \max\{0, p_j - t\} \leq p_i \leq \hat{p}_i \\ \left(\frac{s-p_i}{t}\right)^{1/2} - \frac{1}{2t}p_i\left(\frac{s-p_i}{t}\right)^{-1/2} & \text{if} \quad \hat{p}_i \leq p_i \leq s \\ 0 & \text{if} \quad s \leq p_i; \end{cases}$$

$$MRDU: x = \begin{cases} \min\{1, \left(\frac{s-p_i}{t}\right)^{1/2} - \frac{1}{2t}p_i\left(\frac{s-p_i}{t}\right)^{-1/2}\} & \text{if} \quad p_i \leq \max\{0, s - t\} \\ \left(\frac{s-p_i}{t}\right)^{1/2} - \frac{1}{2t}p_i\left(\frac{s-p_i}{t}\right)^{-1/2} & \text{if} \quad \max\{0, s - t\} \leq p_i \leq s \\ 0 & \text{if} \quad s \leq p_i \end{cases}$$

The inverse demand is given by (2.5) for consumers aware of the existence of both firms and by (2.9) for consumers unaware of firm j. Consider now the case where some consumers are unaware of firm j, say $(1 - k_j)$ of them. The residual demand facing firm i will be:

$$d_i(p_i, p_j, k_i, k_j) = k_i[k_j d_i^a(p_i, p_j) + (1 - k_j)d_i^u(p_i)] \tag{4.1}$$

where $d_i^a(p_i, p_j)$ is given by (2.3) and $d_i^u(p_i)$ is given by (2.8). In figure 5, the demand function (2.3) of the aware is curve DA while the demand function (2.8) of the unaware is curve DU; the associated marginal revenue functions are respectively $MRDA$ and $MRDU$. Although demand curve DU is independent of p_j, the position of demand curve DA depends on p_j; figure 5 is drawn for $p_j = t$ giving a x-intercept at $x = 1$ but

if $p_j < [>]t$ the intercept would be to the left [right] of $x = 1$. Hence for $k_i = 1$:

$$d_i(p_i,p_j,1,k_j) = \begin{cases} 1 & \text{if } p_j > t \quad \text{and} \quad p_i \leq p_j - t \\ k_j DA + (1-k_j)1 & \text{if } \max\{0, p_j - t\} \leq p_i \leq s - t \\ k_j DA + (1-k_j)DU & \text{if } s - t \leq p_i \leq \hat{p}_i \\ k_j DA + (1-k_j)DU = \left(\frac{s-p_i}{t}\right)^{1/2} & \text{if } \hat{p}_i \leq p_i \leq s \\ 0 & \text{if } s \leq p_i, \end{cases} \tag{4.2}$$

and for $k_i < 1$:

$$d_i(p_i,p_j,k_i,k_j) = k_i d_i(p_i,p_j,1,k_j). \tag{4.3}$$

It is clear from (4.3) that the best-reply price p_i to a given (p_j,k_j) is independent of k_i. Hence we can find the best-reply price for $k_i = 1$. It will be convenient and useful to distinguish five cases given the non differentiability of demand functions DA and DU. Considering (4.2) and figure 5, we observe that the best-reply price $p_i^*(p_j,k_j)$ is obtained when $k_j MRDA(p_i,p_j) + (1-k_j)MRDU(p_i) = 0$. Defining $f(p_i,p_j)$ and $g(p_i)$ as follows:

$$f(p_i,p_j) = \frac{p_j + t}{2t} - \frac{1}{t}p_i$$

$$g(p_i) = \left(\frac{s-p_i}{t}\right)^{1/2} - \frac{1}{2t}p_i\left(\frac{s-p_i}{t}\right)^{-1/2}$$

and noting that for $t < s < 3t$, $g(p_i)|_{p_i = s-t} = 1 - \frac{1}{2t}(s-t) \in (0,1)$, then we can identify the following cases (refer to figure 5):

CASE I: $p_i^*(p_j,k_j) < s - t$

This case appears if:

$$[k_j f(p_i,p_j) + (1-k_j)1]_{p_i = s-t} < 0, \tag{4.4}$$

that is if $0 \leq p_j < 2s - t\left(\frac{2+k_j}{k_j}\right)$. In this case, $p_i^*(p_j,k_j)$ is defined by:

$$\left[k_j\left(\frac{p_j+t}{2t} - \frac{1}{t}p_i\right) + (1-k_j)1\right] = 0, \tag{4.5}$$

that is

$$p_i^*(p_j,k_j) = \frac{1}{2}(p_j + t) + \frac{1-k_j}{k_j}t. \tag{4.6}$$

CASE II: $p_i^*(p_j, k_j) = s - t$

This case appears if:

$$[k_j f(p_i, p_j) + (1 - k_j)1]_{p_i = s-t} \geq 0 \tag{4.7}$$

and

$$[k_j f(p_i, p_j) + (1 - k_j)g(p_i)]_{p_i = s-t} \leq 0 \tag{4.8}$$

that is if $\left[2s - t\left(\frac{2+k_j}{k_j}\right)\right] \leq p_j \leq \left[-\frac{3t}{k_j} + \frac{1+k_j}{k_j}s\right]$. In this case, the $p_i^*(p_j, k_j)$ is given by:

$$p_i^*(p_j, k_j) = s - t. \tag{4.9}$$

CASE III: $s - t < p_i^*(p_j, k_j) < \hat{p}_i(p_j)$

This case appears if:

$$[k_j f(p_i, p_j) + (1 - k_j)g(p_i)]_{p_i = s-t} > 0 \tag{4.10}$$

and

$$[k_j f(p_i, p_j) + (1 - k_j)g(p_i)]_{p_i = \hat{p}_i(p_j)} < 0, \tag{4.11}$$

that is if $\left[-\frac{3t}{k_j} + \frac{1+k_j}{k_j}s\right] < p_j < \check{p}_j(k_j, s, t)$, where $\check{p}_j(k_j, s, t)$ is defined as the solution of the following implicit expression:

$$k_j f(\hat{p}_i(\check{p}_j), \check{p}_j) + (1 - k_j)g(\hat{p}_i(\check{p}_j)) = 0$$

In this case, $p_i^*(p_j, k_j)$ is defined implicitly by:

$$\left[k_j\left(\frac{p_j + t}{2t} - \frac{1}{t}p_i\right) + (1 - k_j)\left[\left(\frac{s - p_i}{t}\right)^{1/2} - \frac{1}{2t}p_i\left(\frac{s - p_i}{t}\right)^{-1/2}\right]\right] = 0, \tag{4.12}$$

that is

$$k_j\left(\frac{p_j + t}{2t} - \frac{1}{t}p_i\right) = -(1 - k_j)\left[\left(\frac{s - p_i}{t}\right)^{1/2} - \frac{1}{2t}p_i\left(\frac{s - p_i}{t}\right)^{-1/2}\right];$$

and raising both sides to the power 2, we get

$$k_j^2\left(\frac{p_j + t}{2t} - \frac{1}{t}p_i\right)^2 = (1 - k_j)^2\left[\frac{s - p_i}{t} - \frac{1}{t}p_i + \frac{p_i^2}{4t^2}\left(\frac{t}{s - p_i}\right)\right].$$

A few more manipulations lead to the following 3^{rd}-degree polynomial in p_i:

$$B_1 p_i^3 + B_2 p_i^2 + B_3 p_i + B_4 = 0,$$

where

$$B_1 = 4k_j^2,$$

$$B_2 = 5k_j^2 t - 18k_j t + 9t - 4k_j^2 s + 4k_j^2 - 4k_j^2 p_j,$$

$$B_3 = k_j^2 t^2 - 8k_j^2 st + 24k_j st - 12st + 2k_j^2 p_j t + 4k_j^2 p_j s + k_j^2 p_j^2,$$

$$B_4 = 4k_j^2 s^2 t - k_j^2 st^2 - 8k_j s^2 t + 4s^2 t.$$

CASE IV: $p_i^*(p_j, k_j) = \hat{p}_i(p_j)$

This case appears if:

$$[k_j f(p_i, p_j) + (1 - k_j) g(p_i)]_{p_i = \hat{p}_i(p_j)} \geq 0, \tag{4.13}$$

and

$$[k_j f(p_i, p_j) + (1 - k_j) g(p_i)]_{p_i = \hat{p}_i(p_j)} = g(p_i)|_{p_i = \hat{p}_i(p_j)} \leq 0, \tag{4.14}$$

that is if $\check{p}_j(k_j, s, t) \leq p_j \leq \left[\frac{2}{3}s - t + 2\left(\frac{st}{3}\right)^{1/2}\right] = \overline{p}_j(s, t)$. In this case $p_i^*(p_j, k_j)$ is from (3.4) given by:

$$p_i^*(p_j, k_j) = p_j - t + 2t\left(\frac{s - p_j}{t}\right)^{1/2}. \tag{4.15}$$

CASE V:[9] $p_i^*(p_j, k_j) = \frac{2}{3}s$

This case appears if:

$$g(p_i)|_{p_i = \hat{p}_i(p_j)} > 0 \tag{4.16}$$

that is if $\left[\frac{2}{3}s - t + 2\left(\frac{st}{3}\right)^{1/2}\right] = \overline{p}_j(s, t) \leq p_j$. In this case the optimal $p_i^*(p_j, k_j)$ is given by:

$$p_i^*(p_j, k_j) = \frac{2}{3}s. \tag{4.17}$$

[9] Note that if $s \geq 3t$ then from (3.5), $\left[\frac{2}{3}s - t + 2\left(\frac{st}{3}\right)^{1/2}\right] > s$ and therefore there is no CASE V.

The price reaction function with partial market coverage is illustrated on figure 6; these reaction functions were obtained through numerical evaluation. It should be noted that in comparison with the price reaction function with full market coverage (figure 3), the boundary value for which prices cease to be strategic complements to become strategic substitutes is now $\check{p}_j$ rather than $\tilde{p}_j$, with $\check{p}_j \geq \tilde{p}_j$, and that a new interval appears (CASE II) where prices are strategically independent.

[INSERT FIGURE 6 ABOUT HERE]

The different element of (4.6) and (4.13) may be interpreted as follows: k_j is the probability that a consumer knows firm j; $f(p_i, p_j) = \left(\frac{p_j+t}{2t} - \frac{1}{t}p_i\right)$ is the marginal revenue realised on a consumer aware of firm j; $(1-k_j)$ is the probability that a consumer is unaware of firm j; and the marginal revenue realised on the latter consumer is 1 if $p_i < s-t$ and is $g(p_i) = \left(\frac{s-p_i}{t}\right)^{1/2} - \frac{1}{2t}p_i\left(\frac{s-p_i}{t}\right)^{-1/2}$ if $p_i > s-t$. Therefore, (4.6) and (4.13) are expressions which say that p_i is chosen so that the "expected marginal revenue" is 0. When $k_j = 1$, we have $\left[2s - t\left(\frac{2+k_j}{k_j}\right)\right] = 2s - 3t = \left[-\frac{3t}{k_j} + \frac{1+k_j}{k_j}s\right]$ and $\check{p}_j(k_j, s, t) = \tilde{p}_j(s, t)$; and (4.7), (4.10) and (4.13) collapse as expected to $p_i^*(p_j) = \frac{1}{2}(p_j + t)$. Hence, the five cases are then reduced to (3.6).

From (3.3) and (3.6), and assuming that $t \leq \frac{4}{5}s$, the symmetric Bertrand-Nash full information equilibrium will be on the linear portion of the price reaction curves as in case (a) of figure 4. Moreover, since in CASE I above we have $p_i^*(p_j, k_j = 1) < p_i^*(p_j, k_j < 1)$, then $p_i^*(p_j, k_j)$ is decreasing in k_j and therefore a symmetric equilibrium with $k_i = k_j < 1$ will be higher on the 45°-line in figure 4(a). To analyse the determination of k_i and k_j, we must introduce a cost of information function. Let the cost of informing (a proportion) k consumers be given by $A(k) = \frac{1}{2}k^2$, as in Tirole (1988). The profit maximizing price p_i and market coverage k_i as functions of the other firm's choice (p_j, k_j) are then given for CASE I above by:

$$p_i = \frac{1}{2}(p_j + t) + \frac{1-k_j}{k_j}t \tag{4.18}$$

$$k_i = \min\{1, \frac{1}{a}p_i\left(k_j\frac{p_j - p_i + t}{2t} + (1-k_j)\right)\}. \tag{4.19}$$

Assuming that s is sufficiently large,[10] the symmetric equilibrium $p_i = p_j = p^c$ and $k_i = k_j = k^c$ is obtained [see Tirole (1988)] as:

$$\text{for} \quad a \geq t/2, \quad \begin{cases} p^c &= \frac{2-k^c}{k^c}t &= (2at)^{1/2} \\ k^c &= \frac{2}{1+(2a/t)^{1/2}} \\ \pi^c &= \frac{2a}{(1+(2a/t)^{1/2})^2}; \end{cases}$$

$$\text{for} \quad a \leq t/2, \quad \begin{cases} p^c &= t \\ k^c &= 1 \\ \pi^c &= (t-a)/2. \end{cases}$$

It is interesting to note that if the advertising cost parameter a is high enough to make $k^c < 1$, that is if $a > t/2$, then k^c is decreasing with a and since p^c decreases with k^c then p^c increases with a; both k^c and p^c are increasing in t (recall that t is a coefficient of substitutability: the larger t is, the more differentiated the goods are); and finally π^c is increasing in a and t.[11] On the other hand, if a is small enough to make $k^c = 1$, that is if $a < t/2$, then k^c and p^c are independent of a and p^c increases linearly with t; as for profits, they increase with t but decrease with a: since market coverage is already maximal, an increase in a simply depresses profits. When $a = t/2$ then $(t-a)/2 = \frac{2a}{(1+(2a/t)^{1/2})^2}$. We will use this equilibrium as a benchmark of comparison with the leader-follower equilibrium we characterize in the next section.

Suppose now that firm 1 first chooses its action (p_1, k_1) and firm 2 moves second given that firm 1 is committed to (p_1, k_1) which is observed by firm 2. Given (p_1, k_1), the optimal choice of firm 2 is given by (4.7), (4.10), (4.13), (4.16), (4.18) and from (4.3) by

$$k_2 = \min\{1, \frac{1}{a}p_2 d_2(p_1, p_2, k_1)\}, \tag{4.20}$$

---

[10] We can make this simplifying assumption to characterize the "symmetric" equilibrium although as mentionned previously, we cannot in general make it to characterize the "asymmetric" leader-follower equilibrium.

[11] As noted by Grossman and Shapiro (1984), the fact that π^c is increasing in a is due to the *strategic* nature of market coverage (advertising) in this model: an increase in a reduces advertising and therefore increases the "endogenous product differentiation" and the prices charged by the firms in equilibrium. This effect may explain why some industries or professions encourage restrictions, legal or other, on market coverage by their members.

where $d_2(p_1,p_2,k_1) \equiv d_2(p_1,p_2,k_1,k_2=1)$. Let us assume now that (p_1,k_1) satisfies $p_1 \leq 2s - t\left(\frac{2+k_1}{k_1}\right)$ so that from (4.19), we get

$$p_2 = \frac{1}{2}(p_1+t) + \frac{1-k_1}{k_1}t \tag{4.21}$$

which together with (4.21) characterizes the second mover's best reply to (p_1,k_1). Under the above condition, the best reply price p_2 is less than $s-t$ and therefore the residual demand to firm 2 is the second part of (4.2), that is we can restrict our discussion to that part only. Hence (4.21) can be rewritten as

$$k_2 = \min\{1, \frac{1}{a}p_2\left(k_1\frac{p_1-p_2+t}{2t} + (1-k_1)\right)\}. \tag{4.22}$$

Substituting (4.22) in (4.23) we obtain

$$\begin{aligned} k_2 &= \min\{1, \frac{1}{a}\left(\frac{1}{2}(p_1+t) + \frac{1-k_1}{k_1}t\right)\left(1-k_1+k_1\frac{1}{2t}(p_1 - \frac{1}{2}(p_1+t) - \frac{1-k_1}{k_1}t + t)\right)\} \\ &= \min\{1, \frac{1}{a}\left(\frac{1}{2}p1 - \frac{1}{2}t + \frac{t}{k_1}\right)\left(\frac{1}{2} - \frac{1}{4}k_1 + \frac{k_1 p_1}{4t}\right)\} \\ &= \min\{1, \frac{1}{a}\left(\frac{k_1 p_1^2}{8t} + p_1(\frac{1}{2} - \frac{1}{4}k_1) + (\frac{tk_1}{8} + \frac{t}{2k_1} - \frac{t}{2})\right)\}. \end{aligned} \tag{4.23}$$

Therefore $k_2 > 0$ if $\left(\frac{k_1 p_1^2}{8t} + p_1(\frac{1}{2} - \frac{1}{4}k_1) + (\frac{tk_1}{8} + \frac{t}{2k_1} - \frac{t}{2})\right)$ is positive. This expression is a 2^{nd}-degree polynomial in p_1 whose roots are always negative for all $k_1 \in [0,1]$; since it is equal to $t(k_1-2)^2/2 > 0$ at $p_1 = 0$, we conclude that $k_2 > 0$ for all (p_1,k_1). Moreover, k_2 is non decreasing in p_1 and is therefore minimal at $p_1 = 0$; hence, we get

$$k_2 = \min\{1, \frac{1}{a}p_2\left(k_1\frac{p_1-p_2+t}{2t} + (1-k_1)\right) \leq \frac{1}{2a}t(k_1-2)^2\}. \tag{4.24}$$

From (4.24) we get $k_2 = 1$ if $a \leq t/2$. Since we are interested in small values of a, we will restrict our attention to such "small" a so that $k_2 = 1$ even in the worst case for firm 2 that is $(p_1,k_1) = (0,1)$ in which case we are in CASE I as assumed. This means that in this context the second mover firm always advertize to everyone.

Rather than pursuing the complete characterization of the Stackelberg equilibrium for the general setting, a rather tedious and very cumbersome if at all feasible task, we

ran some numerical evaluation reported in tables 1 and 2.

INSERT TABLES 1 AND 2 ABOUT HERE

Table 1 presents the Stackelberg equilibrium in prices, that is for $k_i \equiv 1, i = 1, 2$. It is interesting to note in table 1 that:

- As product differentiation increases, that is as t increases, the profit of the first-mover firm π_1 is increasing up to $t = 0.85$ and then decreasing while the profit of the second-mover π_2 is increasing up to $t = 0.60$ and then decreasing. Hence, there exists for each duopolists a "best" level of product differentiation: that value is $t = .85$ for the first-mover and $t = .60$ for the second-mover. This suggests that the duopolists would agree on the desirability of more product differentiation up to $t = .60$ but then disagree for intermediate values of t, that is for $t \in (.60, .85)$ and then agree again on the desirability of less product differentiation: when product differentiation is too low, competition is harsh and both firms would agree that it is too low; when product differentiation is high (t is large) then it is too costly for the firms and both would agree that it is too high; for intermediate values, they disagree, the leader would like more differentiation, the follower would want less.

- $\pi_1 < \pi_2$ (followership is the preferred position) up to $t = .80$, that is for low product differentiation; $\pi_1 > \pi_2$ (leadership is the preferred position) afterwards, as long as the market structure remains a duopoly (that is for $t < 1.33$).

- The leader's price (markup) increases with product differentiation up to $t = 0.60$ and decreases afterwards to its monopolized market level $\frac{2}{3}s$ reached for $t = 1$; the follower's price (markup) increases with product differentiation up to $t = 1$ and decreases afterwards to its monopolized market level $\frac{2}{3}s$ reached for $t = 1.33$. This means that markups increase with product differentiation if differentiation is small but decreases with differentiation if differentiation is high. For intermediate values, that is for $t \in (.60, .85)$, one is increasing and the other decreasing.

- The sales levels (the number of consumers served or the number of units sold) of both firms are constant for $t < 0.60$; as differentiation increases above 0.60, the leader's sales increases up to $t = 1$ and then decreases (and inversely for the follower's sales since the sum is 1) as long as the market structure is a duopoly; for $t > 1.33$, the market structure becomes two independent monopolies.

- π_1 and π_2 are both increasing with s up to their maximum value attained for $s = 1.65$.

- $\pi_1 > \pi_2$ (leadership is the more advantageous position) for small values of s, namely $0.75 < s < 1.25$; $\pi_1 < \pi_2$ (followership is the more advantageous position) for large values of s, namely $s > 1.25$.

- For small values of s, namely $s < 0.75$, the equilibrium is the monopoly equilibrium with separate markets and both prices are then equal to $\frac{2}{3}s$.

- Both p_1 and p_2 (the markups here) are increasing with s up to their respective maximum value of 1.5 and 1.25 reached for $s = 1.65$; $p_1 < p_2$ for $0.75 < s < 1.25$ and $p_1 > p_2$ for $s > 1.25$.

Table 2 presents the stackelberg equilibrium in both prices and market coverage. It is obtained for the case $a = 0$ which implies that $k_2 = 1$. It is interesting to note in table 2 that:

- For $t > 0.45$, the Stackelberg equilibrium in prices and market coverages is the Stackelberg equilibrium in prices (that is, $k_1 = k_2 = 1$) and our previous characterization (table 1) holds; for $t < 0.45$ the former equilibrium differs significantly from the latter equilibrium.

- The market coverage of firm 1 is increasing with t up to its maximum value of 1 attained for $t = 0.45$.

- As t increases from 0.20 to 3.00, the profit of firm 1 first increases, attains a maximum for $t = 1$ and then decreases; for $t > 1.33$ the monopoly market size shrinks with

further increases it t; as for the profit of firm 2, it first decreases, reaches a local minimum for $t \approx 0.43$, then increases for a while before decreasing again.

- For large values of t (for significant product differentiation), leadership is the preferred position as $\pi_1 > \pi_2$; for values of t less than 0.80 (for low product differentiation), followership is the preferred position.

- The leader's price (markup) is almost constant (fluctuating in the interval (0.59, 0.62)) for low values of t, that is for $t < 0.45$, and for larger values, it is given in table 1; as for the follower's price, it *decreases* with product differentiation as long as the first-mover chooses a partial market coverage, that is for $t < .45$.

- Up to $s = 2.22$, the Stackelberg equilibrium in prices and market coverages is the Stackelberg equilibrium in prices (that is, $k_1 = k_2 = 1$) and our previous characterization (table 1) holds; for $s > 2.22$ the former equilibrium differs significantly from the latter equilibrium.

- For $s > 2.22$, the market coverage k_1 chosen by the first-mover is decreasing with s.

- π_1 is non-decreasing with s while π_2 is non decreasing with s in the intervals [0.5, 2.22] and [2.23, 6.0]; the profit of firm 2 drops when s goes from 2.22 to 2.23, that is when it begins to be profitable for the first-mover to cover less than 100% of the market.

- For values of s above 1.25 (for reduced product differentiation), and in particular for values of s above 2.22, followership is the preferred position as $\pi_1 < \pi_2$.

- The prices (markups) are non-decreasing with s up to $s = 2.22$, the last value in table 2 for which firm 1 covers the whole market; for $s > 2.22$, the price p_1 drops before increasing again while p_2 keeps on increasing; as s increases above 2.22, the follower's price p_2 increases faster than the first-mover's price p_1.

Clearly these characteristics of the equilibrium when strategies involve both prices

and the choices of market coverage would have significant impact on the equilibrium (perfect) choice of locations in a two-stage model where locations would be chosen simultaneously or sequentially in the first stage.

5. THE RELATED LITERATURE AND CONCLUSION

When a firm (such as the first-mover above) chooses a partial market coverage *even with a zero cost of market coverage or information*, in the sense that being known to more consumers cost nothing, it chooses to restrict the number of its customers. In a sense, this firm is choosing to ration the *potential* consumers by keeping them uninformed of its existence. We showed elsewhere[12] that it may be optimal for a firm to behave in such a way in order to reduce the intensity of the competition it faces: by doing so, it can raise its price above the full market coverage equilibrium level and in spite of the fact that it has fewer customers, it makes more profit (compare table 1 and table 2). There are good reasons for believing that partial market coverage, uncovered in our simple model of a leader-follower spatial duopoly, and more generally "rationing" are indeed present in oligopolistic markets. A first reason is that the results are derived in structural models where all variables are under the control of a well identified and specified agent; this differs from the more usual "reduced-form" approach to spatial oligopoly theory, where the firms are modelled as choosing either a price or a quantity (in the present model, the choice of quantity is done through market coverage since each consumer consumes either 0 or 1 unit of the good) but not both since once one variable is chosen, the other is determined "by the market". It is certainly interesting to verify if the equilibria in structural-form models (where all variables are explicitly chosen

[12] Boyer and Moreaux (1987), (1988), (1989). The first paper considers the case of an homogeneous good duopoly and the other two, the case of differentiated products. In the second paper, we analyse the case where the strategy space of the firms is formed of a price and a production capacity; in the third paper, we consider both the strategy space (price, production capacity) and the strategy space (price, serving capacity) and we compare the two types of equilibria (they are always different). When firms choose a price and a serving capacity (a number of consumers), rationing appears in equilibrium if product differentiation is not too large; when firms choose a price and a production capacity, all equilibria involve a positive level of rationed consumers, and this under very general demand conditions.

by the agents) are similar to the equilibria in reduced-form models. Another reason for being interested in the phenomenon of rationing is that it can be disguised under different names. In the present context of spatial competition, this rationing strategy takes the form of partial market coverage; as mentionned above, this is a way to ration potential consumers, by keeping them uninformed. But this rationing, without queues of unsatisfied consumers, is part of the firm's strategy, that is its efforts to influence the behavior of its competitors. By choosing a partial market coverage, the first-mover is in a sense credibly signaling to the second-mover that he (the first-mover) leaves those uncovered consumers to him (the second-mover); this is likely to make this second-mover a less agressive, more accomodating, competitor allowing the first-mover to raise its price above its otherwise equilibrium level (to a more profitable level). We considered in this paper a particular form of rationing strategy, namely the partial market coverage to which a firm may find profitable to restrict its operations, but there are clearly many others. Examples are: a noncentral location choice, a lower sales effort, a less favorable set of product characteristics; as compared in each case to the non strategic (myopic profit maximising) level of the same variable.

We mentionned in the introduction the papers which to our knowledge endogenizes some form of market coverage variable, although none directly addresses this issue. In a related framework, Grossman and Shapiro (1984) characterize the symmetric Nash equilibrium of a circular city model where n firms, equally spaced around the circle, choose both their price and their informative advertising budget. As in the present context, consumers become informed of the existence (location and price) of a firm when they recieve a message from it (they do not search). The advertising technology is such that it is impossible for a firm to make sure that every consumer did receive a message from it; the cost of advertising goes to ∞ as the market coverage goes to 1; hence, no firm will choose to cover the whole market. Partial market coverage plays no "strategic" – in the sense of modifying the competitors' behavior – role in

Grossman and Shapiro since the firms move simultaneously in a one-shot game. But it is interesting to relate our results to theirs in terms of the relatioships between product differentiation (the number n of firms in their model) and market coverage level (informative advertising in their model), prices, and profits. Grossman and Shapiro showed that market coverage (the informative advertising budget) incrases with product differentiation and that the markup increases also with product differentiation. By making market coverage a strategic variable (by making it endogenous in an asymmetric first-mover/second-mover market structure), we obtained that the first-mover's market coverage (the second-mover's one is always 100%, when it is free) is non decreasing with product differentiation (an increase in t) but markups may increase or decrease with product differentiation: when product differentiation is small ($t < 0.44$), firm 1's markup is constant while firm 2's markup is *decreasing* with differentiation; when product differentiation is intermediate-small ($0.45 < t < 0.8$), the first-mover's markup is increasing but the second-mover's markup is decreasing with differentiation; when product differentiation is intermediate-large ($0.8 < t < 1.0$), markups for both firms are *increasing* with differentiation; finally, when product differentiation is large ($t > 1$), the markups are *decreasing* with differentiation to their separate monopoly market levels $\frac{2}{3}s$ which is reached for firm 1 when $t = 1$ and for firm 2 when $t = 2$.

Grossman and Shapiro also compare the level of informative advertising or market coverage chosen by the oligopolists in the n-firm symmetric Nash equilibrium with its social optimal level. They argue that there are four factors or effects which may cause a discrepancy between the two. The first two effects are due to the fact that oligopolists have market power, leading to lower than optimal market coverage because they concentrate on marginal profit rather than on marginal consumer surplus (too many consumers remain unserved in the oligopoly equilibrium) and on the better matching which more coverage would bring; the third cause is due to the business stealing effect, which has value for the individual oligopolist but has no social value; and finally, the fourth effect

is reduced price elasticity of demand which a lower than optimal market coverage would bring in the case where the individual demand is continuously sensitive to the price (not analysed by Grossman and Shapiro since their model, like ours, assumes a unit demand level if the price is below s). Grossman and Shapiro show that if n is large (product differentiation is low), the symmetric level of market coverage is excessive; for small n, their analysis is inconclusive. In the present paper, we have a case of asymmetric spatial duopolists ($n = 2$) and the socially optimal level of market coverage, under our assumption of no cost of information ($a = 0$), is clearly 100%, that is $k_1 = k_2 = 1$. The Stackelberg equilibrium level of market coverage is indeed optimal if transportation cost t is large (product differentiation is large), but too small if t is small (product differentiation low); it can never be excessive here.

Strategies of partial market coverage by a first-mover firm was interpreted above as a form of rationing the consumers when such rationing is indeed profit maximizing for firm 1. In so doing, the first-mover (the incumbent, or leader) signals to the second-mover (the entrant, or follower) that it does not wish to compete agressively: it is in the best interest of the second-mover to accomodate the first-mover by charging a price above its best reply price under full coverage by the first-mover, that is a higher best reply price in the Stackelberg game in price *and* market coverage than in the Stackelberg equilibrium in price alone (with market coverage fixed at 100%), for any price chosen by the first-mover. In equilibrium, it happens that both prices will be higher in the Stackelberg game in price *and* market coverage than in the Stackelberg equilibrium in price alone. This is a case of a more general phenomenon analysed by Fudenberg and Tirole (1984)[13] but with some important differences. In the Fudenberg-Tirole model, an incumbent monopolist in period 1 is facing a potential entrant in period 2; he can advertize in period 1, at some cost, and create a captive market forever of those consumers who did get a message. If entry occurs, the incumbent and the entrant compete for the remaining

[13] See also Schmalensee (1983), Gelman and Salop (1983), d'Aspremont and Gabszewicz (1985), Henry (1988) and Fershtman and Mueller (1988).

market, but the incumbent cannot price-discriminate between its captive market segment and the duopolistic market. Fudenberg and Tirole show that the incumbent may choose to prevent entry by reducing its advertising level below the level he would choose as a protected monopolist: by so doing, the incumbent announces that he will be more agressive on the uncovered market and therefore depress prices sufficiently (in a perfect equilibrium) if entry occurs that the entrant prefers not to enter. By reducing its captive market size, the incumbent monopolist avoids being a "fat cat" in period 2.

In the present case of a single-period spatial competition model with a first-mover and a second-mover, we noted already that the preferred position need not be leadership; whether it is or not depends crucially on the level of product differentiation. When the preferred position is followership (product differentiation is low), then it is probable that the dominant firm will in fact be the second-mover rather than the leader.[14] If that is the case, then the entrant will take the position of the first-mover and by choosing a partial market coverage make the dominant first-mover a "fat cat" accomodating the first-mover: the roles are reversed, somewhat in the spirit of Gelman and Salop (1983).

[14] See Boyer and Moreaux (1987).

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FIGURE 1

THE MARKET COVERAGE

for $k_i = k_j = 1$

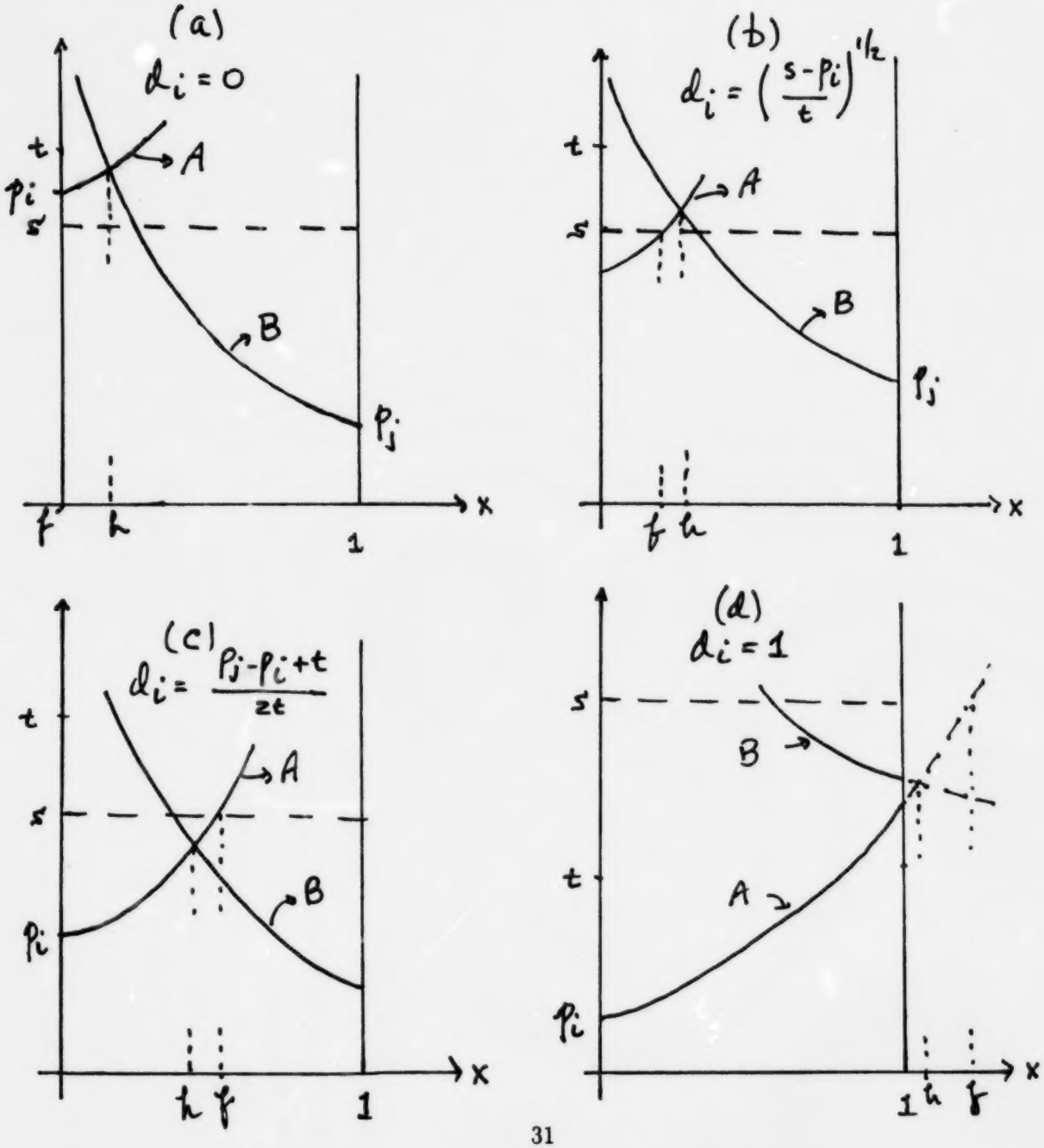

FIGURE 2

THE DEMAND FUNCTION

for $t > s$ and $p_j < t$

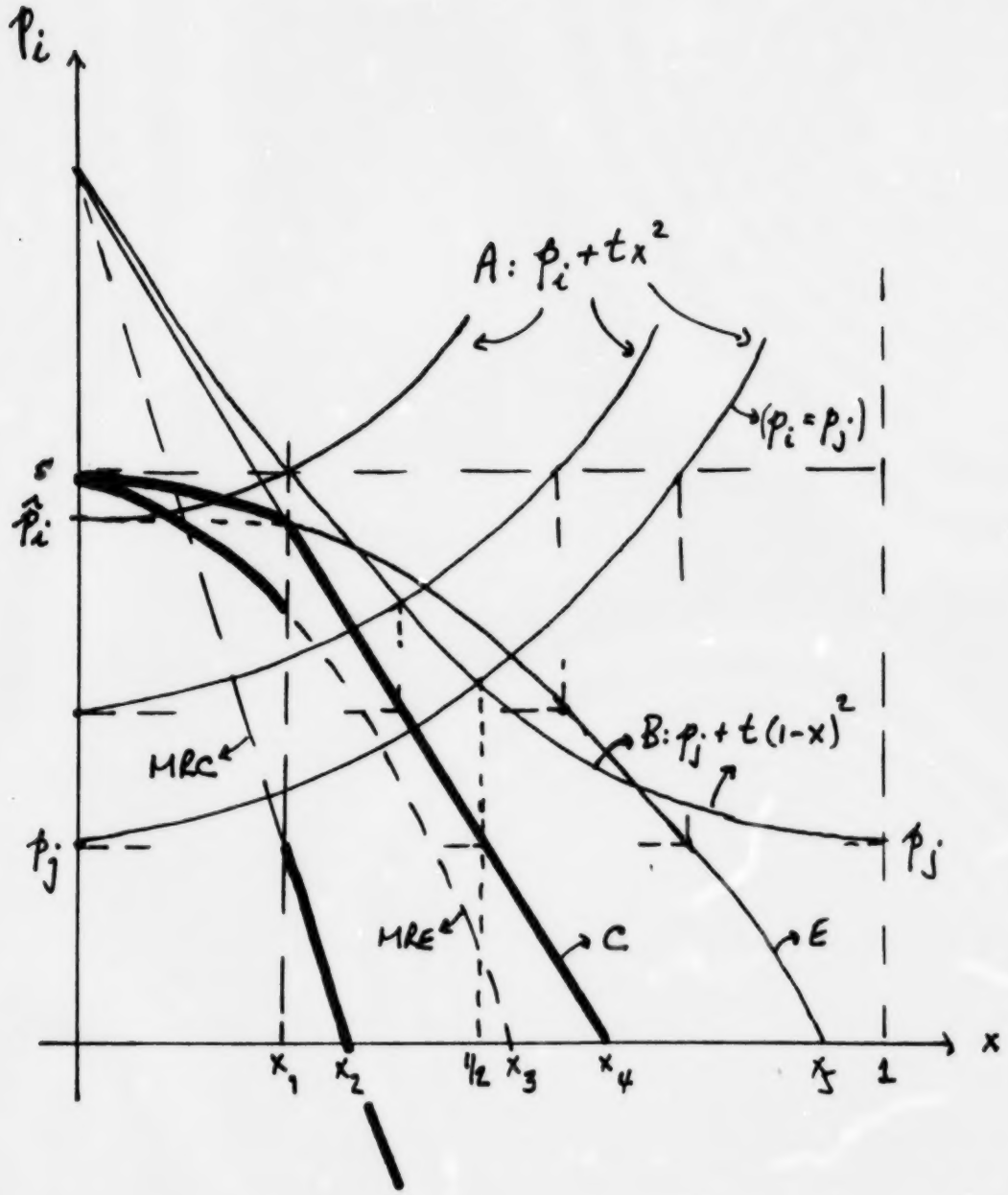

FIGURE 3

THE PRICE REACTION FUNCTION
WITH FULL INFORMATION (COVERAGE)

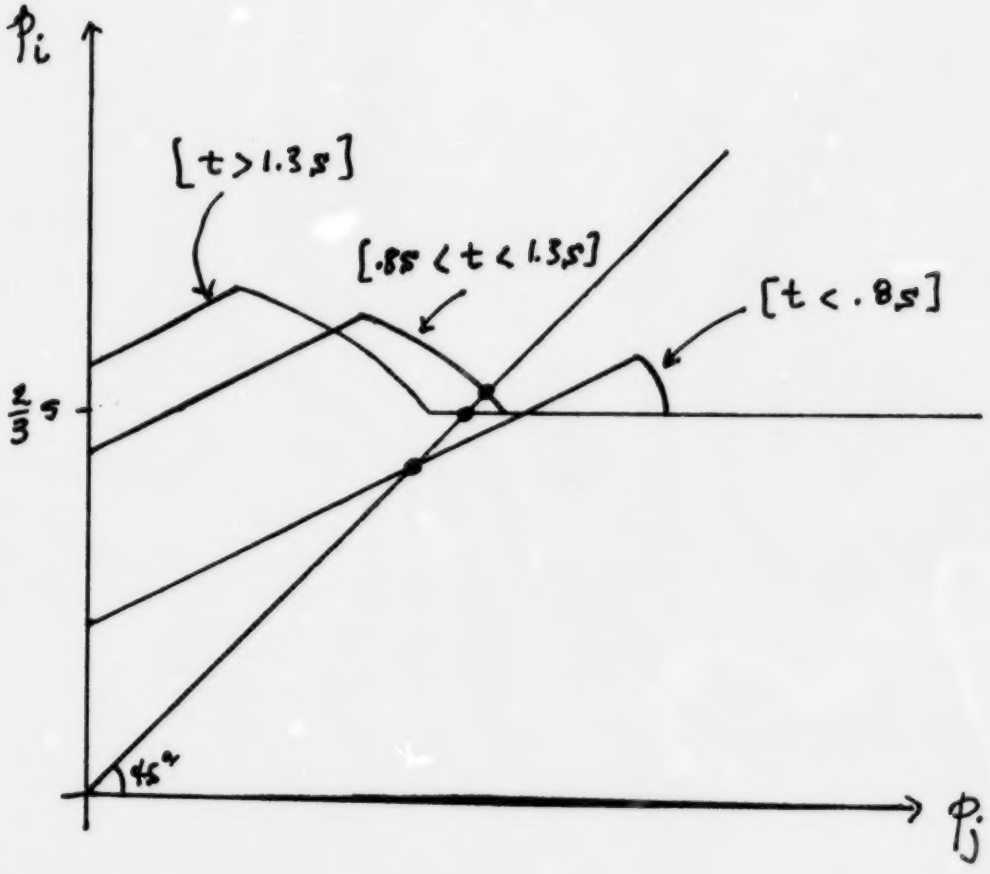

FIGURE 4

THE BERTRAND-NASH EQUILIBRIA

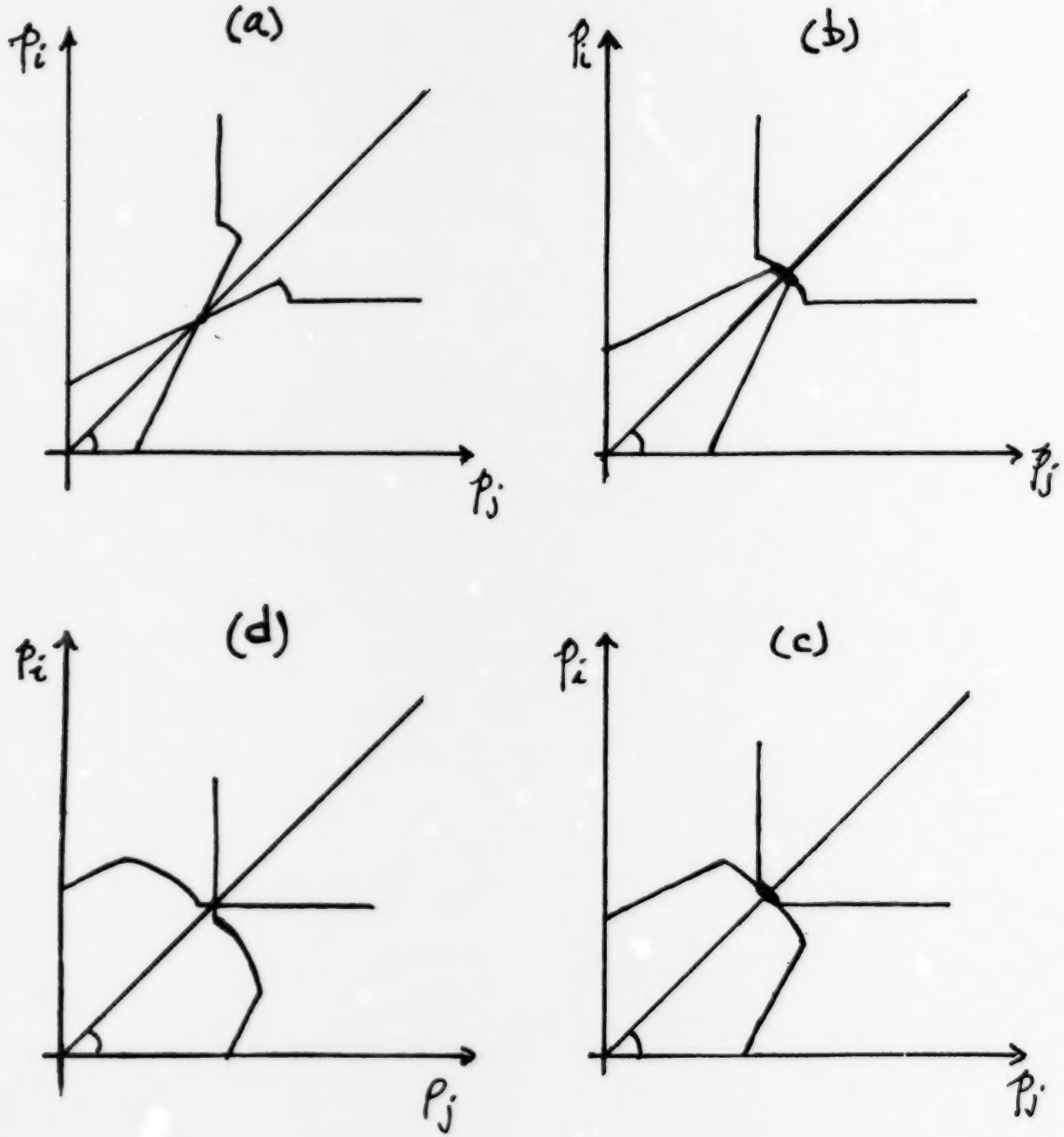

FIGURE 5

THE DEMAND FUNCTION

for $\frac{1}{3}s < t < \frac{4}{5}s$

(that is, for $\frac{5}{4}t < s < 3t$)

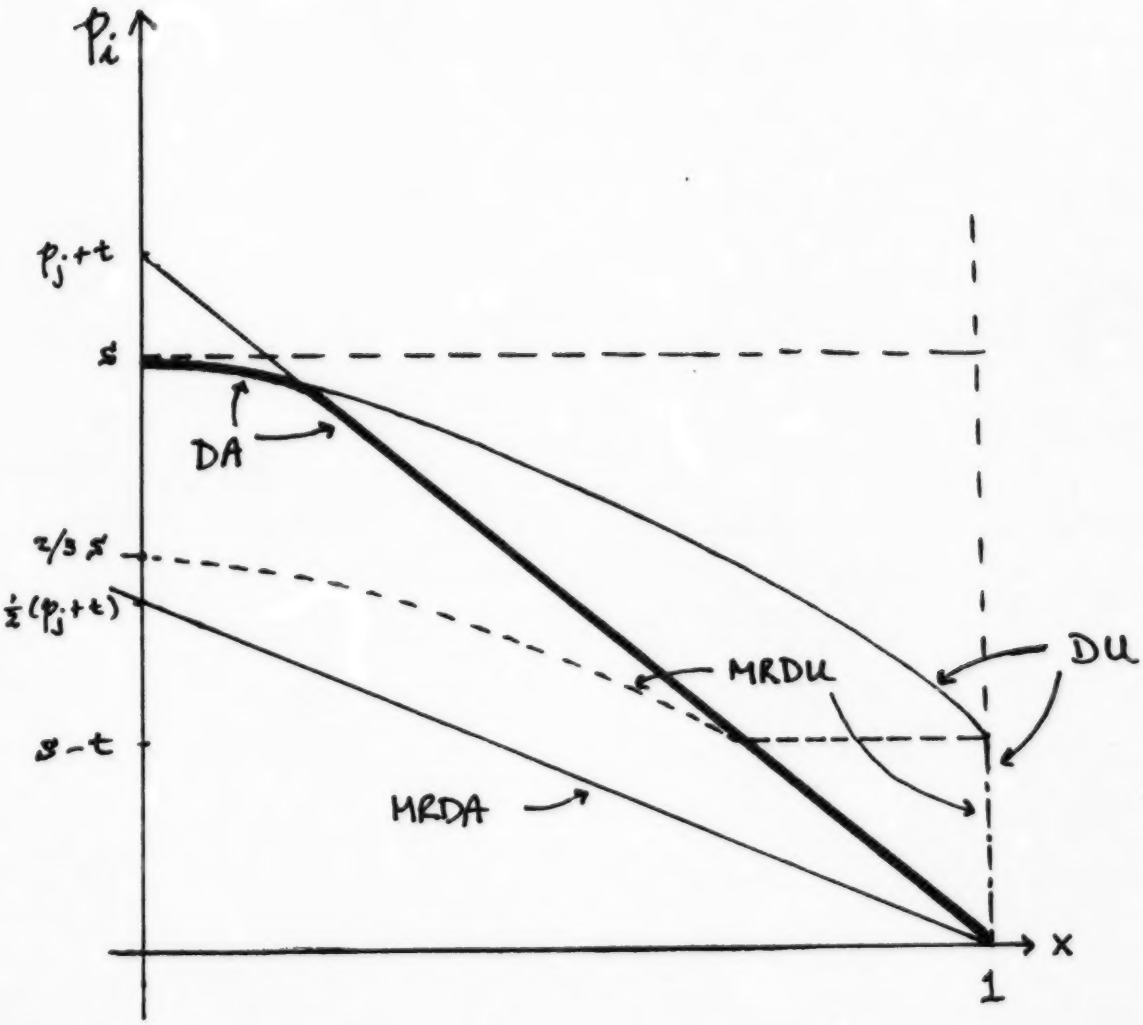

FIGURE 6

THE PRICE REACTION FUNCTION

WITH PARTIAL INFORMATION (COVERAGE)

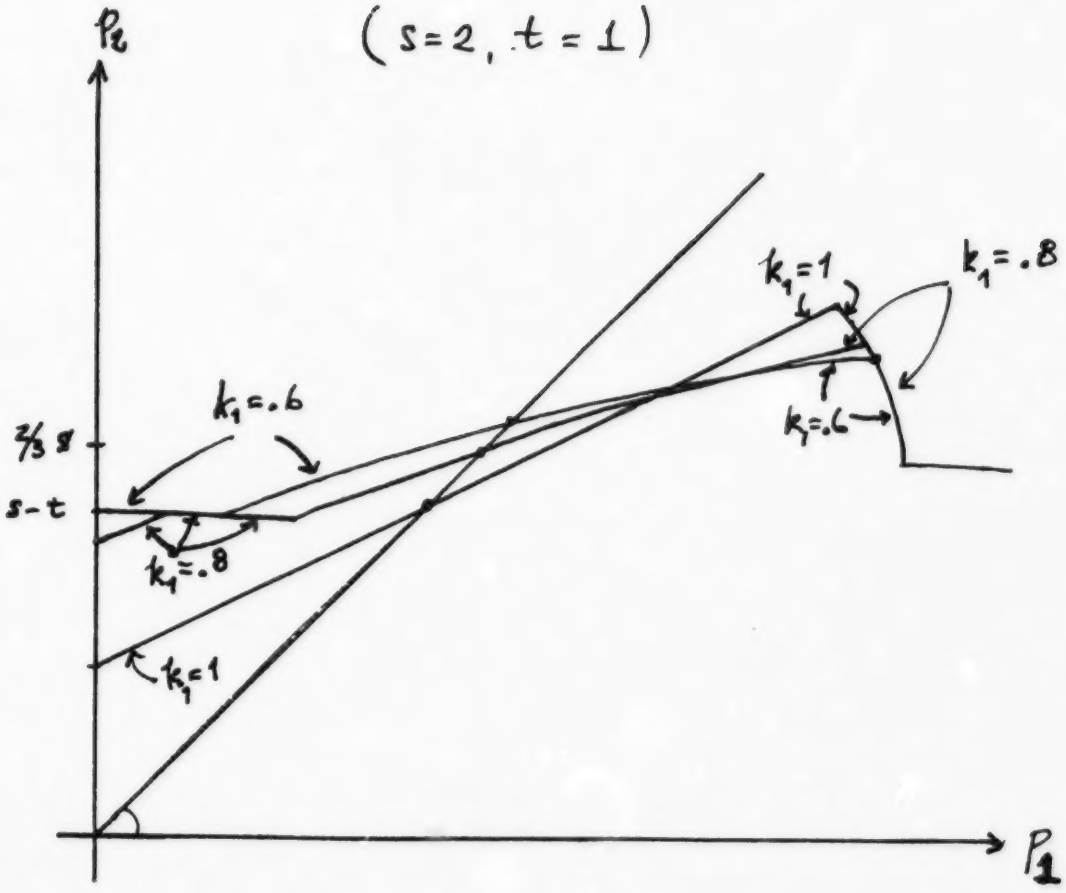

TABLE 1

THE NUMERICAL EVALUATION OF THE STACKELBERG EQUILIBRIUM IN PRICES

for variable t ; $s = 1$ and $a = 0$

t	π_1	π_2	p_1	p_2	D_1	D_2
0.20	0.113	0.156	0.30	0.25	0.38	0.62
0.30	0.169	0.234	0.45	0.37	0.38	0.62
0.40	0.225	0.312	0.60	0.50	0.38	0.62
0.45	0.253	0.352	0.67	0.56	0.38	0.62
0.50	0.281	0.391	0.75	0.62	0.38	0.62
0.55	0.309	0.430	0.82	0.69	0.38	0.62
0.65	0.363	0.456	0.89	0.77	0.41	0.59
0.70	0.381	0.435	0.86	0.78	0.44	0.56
0.75	0.393	0.416	0.83	0.79	0.47	0.53
0.80	0.400	0.400	0.80	0.80	0.50	0.50
0.85	0.402	0.384	0.76	0.81	0.53	0.47
1.00	0.385	0.346	0.67	0.82	0.58	0.42
1.25	0.344	0.342	0.67	0.71	0.52	0.48
1.33	0.333	0.333	0.67	0.67	0.50	0.50
2.00	0.272	0.272	0.67	0.67	0.41	0.41

for variable s ; $t = 1$ and $a = 0$

s	π_1	π_2	p_1	p_2	D_1	D_2
0.50	0.136	0.136	0.33	0.33	0.41	0.41
0.75	0.250	0.250	0.50	0.50	0.50	0.50
0.80	0.275	0.273	0.53	0.57	0.52	0.48
1.00	0.385	0.348	0.67	0.82	0.57	0.43
1.20	0.481	0.466	0.93	0.96	0.52	0.48
1.25	0.500	0.500	1.00	1.00	0.50	0.50
1.30	0.514	0.530	1.06	1.03	0.49	0.51
1.50	0.554	0.673	1.32	1.16	0.42	0.58
1.65+	0.563	0.781	1.50	1.25	0.38	0.62

TABLE 2

THE NUMERICAL EVALUATION OF THE STACKELBERG EQUILIBRIUM IN (P,K)

for variable t ; $s = 2$ and $a = 0$

t	k_1	π_1	π_2	p_1	p_2	D_1	D_2	FID_2
0.20	0.34	0.199	0.542	0.62	0.80	0.32	0.68	0.05
0.25	0.44	0.208	0.493	0.61	0.79	0.34	0.66	0.22
0.35	0.68	0.229	0.410	0.62	0.65	0.37	0.63	0.46
0.40	0.82	0.240	0.369	0.62	0.60	0.38	0.62	0.54
0.42	0.88	0.245	0.352	0.62	0.58	0.39	0.61	0.56
0.44	0.94	0.250	0.333	0.62	0.56	0.40	0.60	0.57
0.45	1.00	0.255	0.352	0.68	0.57	0.38	0.62	0.62
0.50	1.00	0.285	0.391	0.75	0.63	0.38	0.62	0.62
0.80	1.00	0.400	0.400	0.80	0.80	0.50	0.50	0.50
1.00	1.00	0.385	0.346	0.67	0.82	0.58	0.42	0.42
1.20	1.00	0.351	0.347	0.67	0.73	0.52	0.48	0.48
2.00	1.00	0.272	0.272	0.67	0.67	0.41	0.41	0.41
3.00	1.00	0.222	0.222	0.67	0.67	0.33	0.33	0.33

for variable s ; $t = 1$ and $a = 0$

s	k_1	π_1	π_2	p_1	p_2	D_1	D_2	FID_2
1.65	1.0	0.563	0.781	1.50	1.25	0.38	0.62	0.62
2.22	1.0	0.563	0.781	1.50	1.25	0.38	0.62	0.62
2.23	0.97	0.564	0.733	1.40	1.23	0.40	0.60	0.58
2.25	0.95	0.566	0.739	1.39	1.25	0.41	0.59	0.57
2.30	0.92	0.573	0.777	1.43	1.30	0.40	0.60	0.56
2.40	0.87	0.587	0.852	1.50	1.40	0.39	0.61	0.54
2.50	0.82	0.601	0.922	1.56	1.50	0.38	0.62	0.53
2.75	0.73	0.637	1.110	1.74	1.75	0.37	0.63	0.50
3.00	0.63	0.675	1.250	1.80	2.00	0.38	0.62	0.40
4.00	0.43	0.831	1.910	2.29	3.00	0.36	0.64	0.15
6.00	0.30	1.102	3.381	3.83	4.75	0.29	0.71	0.04